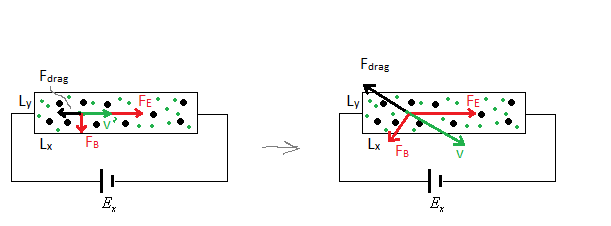
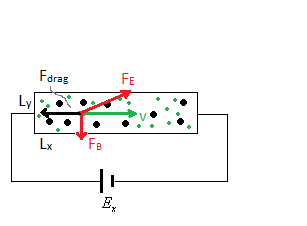
**Classical Model of Hall Conductivity**

So here’s the typical Hall setup. We apply a horizontal voltage /electric field in the x-direction, and a magnetic field in z direction (if there were no impurities, the electrons would eventually execute a kind of curly cue motion with average drift velocity **v** = (E/B) ×, but the impurities prevent the electrons from getting to this point and we’ll find a component of the drift velocity in the direction of **E** and in the direction of **E**×**B**). This is how that comes about. Say our charges are positive, for illustration, and are accelerated to the right by E. As they pick up speed, the magnetic field deflects them downward. And then the drag force pulls them back in the direction of the velocity. So have something like this. Initial situation is on left, and then the temporary steady-state situation is on right. Note **F**E + **F**B = -**F**drag in the steady-state guy.



But then, thanks to the angled **v**, presumably (positive, in our illustration) charge accumulates on the bottom, and a dearth on the top, setting up an upward electric field Ey, which will push **v** more towards the top of the page. I guess the charge density imbalance will wax, along with the consequent Ey field, until the drag velocity has become purely horizontal again (then obviously we’ll get no more charge density deposition). Then we’ll have this situation:



Okay, now want to be quantitative.

**2D Conductivity/Resistivity Tensor**

We have a particle moving in an electric and magnetic field **E**, **B**, subject to dissipative collisions. Consider a **B** field in the z direction, and **E** restricted to the xy plane. Recall that the drift velocity, v is in the direction E×B, when no impurities are present. Anyway, going back to our Boltzman equation, or a version of it:



the Drude model for the motion is as follows.



(e is a signed quantity) If we’re interested in the drift velocity of the charges, then we’d set the d**v**/dt term to zero. And solving for **v**, we’d have:



which amounts to the following equation:



And can invert,



This translates into the following matrix equation (where ωB = |e|B/m):



Can write this as:



and finally,



Let’s look at the magnitude of v.



So interestingly, the stronger B is, the smaller the speed becomes. So the velocity doesn’t *just* change direction, it also diminishes. We’ll also observe the speed in the E direction diminishes. And also, a velocity develops in the **E**×**B** direction regardless of sgn(e). The magnitude of the speed will increase with B for a while, and then diminish. Note if we had a free particle, τ → ∞, then we’d have:



which is what we found in the Electrodynamics folder when investigating the motion of a charged particle in an EM field, and in the Free Day folder. In the limit of strong damping, this would come out to:



Let’s work out the conductivity and resistivity tensors…note all densities are surface densities, since we’re dealing with 2D stuff. Now just to be clear, the current in the, say, y direction is charge times number of particles crossing the y surface (which has length Lx) per unit time, so:



where surface current density jy = nevy. And likewise for jx = Ix/Ly, etc.. So,



Multiplying both sides by ne (again, n is surface density really),



and so we see,



where by σDC I just mean the usual Drude value, not to suggest we’re not technically DC here too. And since **E** = ρ**j**, we have the resistivity matrix **ρ** = **σ**-1 of course,



which is,



and ρDC = m/ne2τ = 1/σDC. Note that the off diagonal elements of the resistivity tensor are independent of the scattering time. How do we interpret the tensor elements? Explicitly…



and,



So σxx diminishes for instance, because the magnetic field diminishes and reorients the drift velocity so that we get smaller vx for the same Ex’ now. But it doesn’t change ρxx because setting jy = 0 requires adding an Ey field component which cancels the magnetic force (making the green arrow go horizontal), and effectively returns us to the non-B situation.

With those formulas above, let’s consider a more direct approach to the coefficient ρyx, which is related to the Hall coefficient. So,



This just shows that the resistivity and resistance have the same units, and are the same, in 2D. Continuing, we have:



now the velocity vx that would result in no net vy would be the one such that FE(y) + FB(y) = 0 → eEy - evxB = 0. So filling in our result for vx = Ey/B, we have:



We should have ρxy = ρyx and so we could evaluate this same resistivity via:



which is indeed the same result. Might stop to note that the transverse velocity comprising the current jy is vy = -Ex/B = (1/B2)**E**×**B** = vdrift (see EM folder). This is the velocity expectation of a charged particle in a crossed **E**, **B** field (and no impurities). Not sure why it should be that vy = vdrift but it is. Anyway, so we have:



where RH is our Hall coefficient. Maybe writing it as:



Makes the proportionality with B easier to understand. So the transverse voltage that’s set up is proportional to the B field and the longitudinal current (density), which kind of makes sense from a N2L perspective. This equation says that if we increase the current in the x-direction, then the voltage difference in the y-direction will increase. This makes sense because increasing the velocity vx of the charges will increase the deflection force from B, which will increase the charge density deposition on the surface of the conductor, which will increase the electric field Ey, and then the voltage as well, until a force balance is achieved. Moreover, the increase in the potential is proportional to the strength of the magnetic field, B, which makes sense because B is proportional to the deflecting magnetic force.

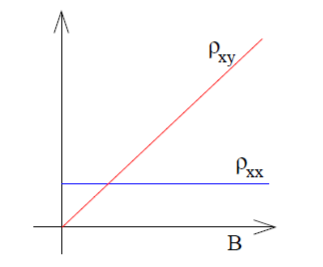
We’ll notice that RH is proportional to the sign of the charge carriers, which provides a way to distinguish + from – charge flow, because note regardless of sign, the charges will be pushed downwards (and so if charge carriers are positive/negative this will correspond to positive/negative current downward). Also important to note that the Hall resistivity doesn’t have anything to do with impurities, as τ doesn’t show up here at all. And that’s because it isn’t the impurities which are providing the resistance to the current in the y direction; rather it’s the transverse electric field Ey which acumulates and impedes the flow of current in the y direction (so I could have put this in the Free Electrons folder). And let’s go for ρxx too:



So,



If we plot these two as a function of B, we should get:



I’ll throw in another definition. The magnetoresistance of a conductor in a crossed electric + magnetic field is defined as the ratio of **E** along **j**, to j:



So let’s work this out. Go back to:



Magnitude of **j** is:



Thankfully, the magnitude of **j** doesn’t depend on the direction of **E**. So,



So,



Huh. Well that’s underwhelming. This seems odd because if τ → ∞, then the drift velocity would be perpendicular to **E**, and so ρB would be zero. Well that’s okay because in the limit τ → ∞, ρB does → 0, since ρDC = m/ne2τ.

**Time Reversal Symmetry**

Note that the following identity is preserved:



which is a consequence of TRS.

|  |
| --- |
| As a practical matter, it seems that the Hall effect is a large B phenomenon, and maybe not ‘covered’ in standard scaling theory?? |

**3D Conductivity/Resistivity Tensor?**

If we do it for arbitrary **v**, **B**, **E**, then we get the following result, though my – sign doesn’t reduce to the previous case – maybe I should have transpose of this??



In any event we have



Note that if we had E and B both pointing in the x direction, say, then we’d get:



which indicates that there would not be a reduction in the conductivity in this direction, since Bx = B. Nor would there be current in the other two directions since Ey = Ez = 0. So basically, nothing would happen.